The Influence of Thermal Convection on Density Segregation in a Vibrated Binary Granular System

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Using a combination of experimental results and discrete particle method simulations, the rôle of buoyancy-driven convection in the segregative behaviour of a three-dimensional, binary granular system is investigated. A previously unobserved relationship between convective motion and segregation intensity is presented, and a qualitative explanation for this behaviour is proposed. This study also provides a first insight into the rôle of diffusive behaviour in the segregation of a granular bed in the convective regime. The results of this work strongly imply the possibility that, for an adequately fluidised granular bed, the degree of segregation may be indirectly controlled through adjustment of the system's driving parameters, or the dissipative properties of the system's side-boundaries.

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I. INTRODUCTION

Granular materials have been extensively studied for more than two centuries due in part to the plethora of interesting and unusual phenomena they exhibit [1] but also to their relevance in a variety of industrial settings [2] and their importance in geophysical phenomena [3–5]. Granular materials are distinct from classical materials due to the innately dissipative interactions between their constituents [6], giving rise to many of the behaviours observed in granular materials that have no equivalent in classical materials. One such phenomenon is *granular segregation*, whereby bi- or poly-disperse granular systems may spontaneously separate into their individual components [7]. Despite extensive research [8], the wide variety of factors influencing the degree to which a system exhibits segregation means that we are still far from fully understanding the phenomenon [9]. Such factors include differences in particle size [10], density [11] and inelasticity [12], the number of particles in the system and the relative concentration of each particle type [13], and the strength with which the system is driven [14]. More recently, thermal diffusion has been shown to have significant impact on segregative phenomena in strongly driven systems [15–17]. However, relatively little experimental work has been performed in this area [18]. One subject which has received relatively little direct attention from the scientific community is the influence of *convection* on segregation due solely to differences in particles' material properties. 'Thermal' granular convection [52] is in many ways analogous to Rayleigh-Bénard convection in classical fluids [19]. For granular systems, where the energy scale of the 'normal', thermodynamic temperature is negligible compared to the kinetic energies of the macroscopic particles involved, we define a 'granular temperature', T, based on the fluctuation of particle velocities about a mean value [53]. For a granular bed excited by, for instance, a vibrating plate at the base of the system, one observes the spontaneous formation of a vertical T-gradient due to dissipative interactions within the bed which may, in turn, lead to convective motion. If a granular system is housed in a container with lateral boundaries which are adequately dissipative compared to the bulk of the system, increased relative energy loss at the walls will lead to a locally increased density and decreased T [21] hence, due to buoyancy effects [20], ensuring that flow is always oriented downward at the walls and upward in the centre of the system [33], and thus providing a steady state. The aim of this paper is to directly investigate the effect of convection on segregation in a binary granular system whose components are equally sized but differ in their densities and dissipative properties. The results of this work not only highlight the highly significant rôle of convective motion in the segregation of such mixtures, but also suggest the possibility that the degree to which such a system exhibits mixing or segregation can be controlled by altering convection strength within the system. It has been shown previously [24] that the strength of the convection discussed above can be tuned through adjustment of the dissipative properties of the system sidewalls. Hence, one can potentially control the degree of segregation within a granular system simply by altering its wall material, without the necessity of changing the system geometry, the composition of the granular bed, or the method by which it is driven. Clearly, this could prove extremely useful in various industrial applications where segregation may be undesirable, or indeed may be required [25].

II. EXPERIMENTAL DETAILS

A. System Details

The main experimental system consists of a granular bed of 500 glass and 500 steel beads, each of 5 ± 0.1 mm diameter, housed in a square-based container of height $H = 200 \pm 1$ mm and width $W = 100 \pm 1$ mm. Additional experiments are also conducted using equivalent numbers of glass and brass spheres, as well as monodisperse systems of $N = 1000$ glass or steel beads. H is adequate to minimise particle collisions with the upper boundary, and its width and depth are such that the system can be considered fully 3D. The relatively large particle size means that interstitial air effects, which may affect segregation phenomena [26], can be neglected [27]. The system walls are interchangeable, allowing the degree of side-boundary dissipation and hence convection within the system to be altered. A complete list of sidewall materials used alongside their respective *effective elasticities*, ε_w, can be seen in table I. ε_w provides a measure of the average change in translational energy during a particle-wall collision due to both normal restitution *and* rotational motion. $\varepsilon_w = 1$ corresponds to a perfectly elastic collision, while $\varepsilon_w = 0$ defines a perfectly dissipative collision. The quoted values correspond to collisions between glass particles and the relevant sidewall material, as measured experimentally in [24]. Although values will vary slightly for steel beads, for the purposes of this paper ε_w serves simply as a measure of the relative elasticity of each material, such that general qualitative trends can be observed. The system is vibrated sinusoidally in the vertical direction, imparting energy to the granular bed through particle collisions with the container base. Oscillations of fixed frequency $f = 70$ Hz and amplitude $A = 1.17$ mm provide a fully-fluidised bed. Since vibration is a common method employed in industry for the mixing of granular materials [28], this method of excitation seems appropriate.

TABLE I: Effective elasticities for particle-sidewall collisions for the various wall materials used in experiment.

Material	Effective Elasticity, ε_w
Mild Steel	0.70 ± 0.006
Copper	0.58 ± 0.008
Brass	0.52 ± 0.010
Tufnol	0.39 ± 0.012
Clear Perspex	0.33 ± 0.014

B. Positron Emission Particle Tracking

Data is acquired using positron emission particle tracking (PEPT), a non-invasive technique whereby the time-averaged behaviour of a single 'tracer' particle in a steady-state granular system can be used to extract information regarding the behaviour of the system as a whole. The tracers used are physically identical to the other particles in the system, aside from the fact that they are 'labelled' with a positron-emitting radioisotope. The β^+ particles emitted from these isotopes rapidly annihilate within the tracer material, causing the emission of a pair of gamma-rays whose trajectories are separated by 180°. Upon placing the tracer between the detectors of a dual-headed gamma camera, these back-to-back gamma-rays can be used to triangulate the position of the particle multiple times a second, and thus record its motion through a system. Particle motion is recorded in three dimensions with a spatial resolution of up to 1mm and a temporal resolution on the millisecond scale [29]. For ergodic, steady state systems such as the one detailed here, the long-time average of a single particle's behaviour can be used to extract information pertaining to the system as a whole. For binary and polydisperse systems, individual runs are conducted using tracers of each individual species. The data acquired can then be combined to provide information relevant to the system as a whole. PEPT can be used to determine a multitude of important quantities, including one-, two- and three-dimensional density and temperature fields [30, 35], mean squared displacements and diffusion coefficients [32], convection strengths [24, 33] and, in bi- and poly-disperse systems, individual particle distributions and segregation intensities [34]. Full details regarding the PEPT technique can be found in references [29, 32], and further information regarding its application to binary systems in reference [35].

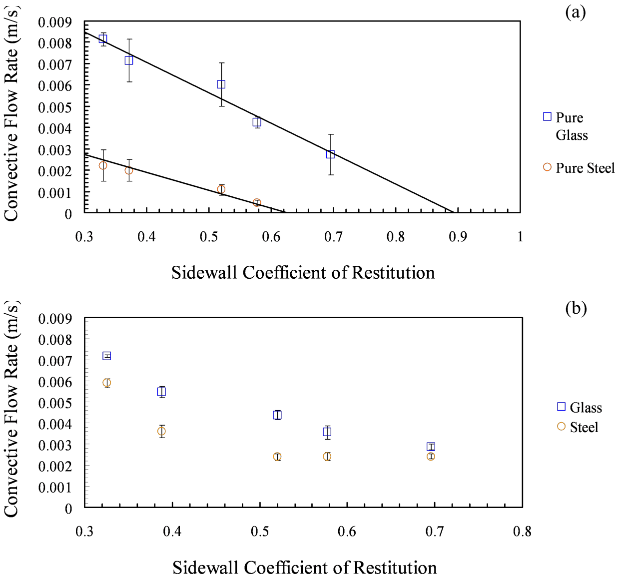

FIG. 1: Variation of convective flow rate with ε_w for (a) monodisperse systems of glass and steel spheres and (b) a bidisperse mixture of these materials.

III. RESULTS AND DISCUSSION

Figure 1 shows the variation of convective flow rate, J with ε_w for each particle type. J is determined as the average particle velocity through the vertical centre of convection for the system, where the horizontal components of velocity are zero [36]. Specifically, $J = \frac{\sum_i^n |v_z^i|}{2n}$, where v_z^i is the vertical component of velocity corresponding to the i^{th} location event and n is the total number of relevant data points [54]. Figure 1a compares the observed behaviour for *monodisperse* systems of each particle type. In each case, $N = 1000$. Figure 1b shows the behaviour for the steel and glass components of a single, bidisperse mixture with 500 of each species, thus keeping the total N fixed at 1000. The transition from a monodisperse to a binary system shows some interesting effects on the convective behaviour of the bed. It is firstly notable that, for the glass component of the binary system, J is slightly reduced compared to the pure-glass system, whereas for the steel component J becomes considerably larger. This change can perhaps be explained by differences in elasticity between the two components - steel particles are more dissipative than their glass counterparts [37]. Thus the average dissipation in a mixed system will be higher than that of a system composed entirely of glass particles, yet lower than for a pure-steel system, leading to the observed changes in flow rate. This difference in particle elasticity is also believed to explain the considerably reduced x-intercept observed for the monodisperse steel system compared to the glass system [24]. The second noteworthy feature is the apparent 'plateau' in convection rate observed in the steel component of the binary system as ε_w increases implying that, unlike the case of monodisperse beds, such a system may possess an inherent minimum value of J. Further research is required in order to verify and provide an explanation for this interesting observation. Having discussed the manner in which ε_w affects the convective flow rate of a granular system, we now address the question of how J, in turn, influences segregation within said system. The degree to which a binary granular system undergoes segregation or mixing can be quantified in a manner analogous to that of [38] by dividing the experimental system into a series of cells and calculating the intensity of segregation as:

$$I_s = \left[\frac{\sum_{i=1}^{i=N_c}(\varphi_i - \varphi_m)^2}{N_c}\right]^{\frac{1}{2}} \tag{1}$$

Here, N_c is the total number of cells, φ_i is the single species concentration in the i^{th} cell, and φ_m is the system's mean concentration. A value $I_s = 0$ corresponds to a perfectly mixed system while $I_s = 0.5$ indicates complete segregation. The values of I_s discussed in this paper correspond to the *equilibrium distribution* reached by the system. In order to confirm that, for all data sets, the system is in indeed in a steady state, each 3600s run is divided into a series of time intervals. I_s is then calculated for each of these windows, and the steady state taken as the point in time at which variations in I_s become negligible. In fact, the evolution of the system towards equilibrium is extremely rapid ($\Delta t_{seg.} \ll 3600$ s).

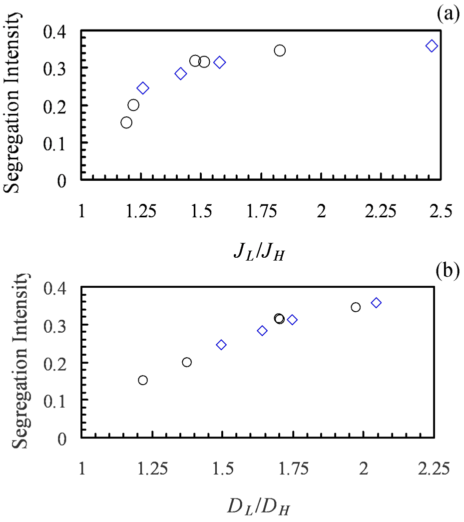

FIG. 2: (a) Segregation intensity as a function of the ratio of convective flow rates for light (glass) and heavy (steel) particles. Variation in the value of J_L/J_H is induced by altering either ε_w (circles) or the system's driving parameters (diamonds). (b) variation of I_s with the ratio of the self-diffusion coefficients, D, for the two species of particle.

Interestingly, I_s displays no clear, monotonic dependence on the overall convective flow rate, as might be expected from previous studies [11, 39]. Neither is there any clear dependence on either of the individual J-values for the separate species; rather, it seems that the key parameter is the *difference* in J between species. Figure 2 shows the variation of I_s with the ratio of convective flow rates for the two components of the system, demonstrating the *significant degree* to which convective behaviour can alter the level of segregation.

The rôle of the differential convection, J_L/J_H, between particle species is more clearly illustrated in Figure 3. The connection between this differential convection and I_s can be understood on a qualitative level by considering the fact that denser, and hence heavier, particles are less likely to be dragged into the convective stream [40]. For systems in which convection is weak (Figure 3 (a)), the implicitly small difference in convection rate between the two species leads to a correspondingly small degree of segregation. Conversely, for the case of very strong convective flow, the likelihood of heavy particles being 'swept up' in the convective flow increases, leading once again to similar flow rates for each species, and hence a reduced degree of segregation (Figure 3 (b)). The strongest segregation occurs in the mid-range between these two extremes, where lighter and less dissipative particles undergo significant convective motion, while heavier and less elastic particles remain relatively unperturbed (Figure 3 (c)). It is interesting to note the stark contrast of these findings to previous studies, which consider only a system's average convection rate and suggest a monotonic relation between this value and the observed degree of segregation. Here, however, we observe the strongest and weakest *whole-system* convection rates, J_{tot}, to demonstrate highly similar degrees of segregation, while the degree of segregation observed for intermediate J_{tot} values can be more than a factor of two higher! Such a finding clearly illustrates that future studies of binary and polydisperse systems must more carefully consider the precise behaviour of convective flow. Moreover, it is also notable that in the three cases described above, the systems are *identical* in every aspect other than the material of the walls bounding the system. Thus, Figure 3 provides a direct demonstration of the ability to control the degree of segregation within a system solely through alteration of wall material.

It is perhaps worth noting that, in all cases, segregation occurs predominantly in the vertical direction. However, a small degree of horizontal segregation is also observed, with lighter, less dissipative particles showing a slight tendency to cluster near the walls of the container. This behaviour may be simply explained by the greater difference in dissipation between interparticle and particle-wall collisions for more elastic species. The fact that the effect is slightly more pronounced in systems with more dissipative sidewalls lends credence to this hypothesis. Nonetheless, even in the most extreme examples, this horizontal component of I_s is an order of magnitude smaller than the vertical, making it a decidedly secondary effect. In order to verify that convection, and not other sidewall effects, is indeed the dominant factor producing the observed behaviour, additional data was obtained keeping ε_w constant and instead altering the convective flow rate of the system by varying the driving parameters. The ranges of driving frequency and amplitude used - $f \in (50, 90)$ Hz, $A \in (0.66, 1.69)$ mm - were chosen to ensure variation not only in f and A but also driving velocity, V, and acceleration, Γ. The collapse of all data points onto a single, monotonically-increasing curve suggests that the degree of segregation in the system is not strongly dependent on the driving parameters *or* specific effects due to sidewall dissipation, supporting the idea that convection is indeed the primary criterion affecting I_s. Moreover, the lack of correlation observed between I_s and the system's density and temperature (which also vary as V and ε_w are altered) implies a lack of dependence on these parameters also. It should be noted, however, that this independence can only be expected to hold in collisionally-dominated systems such as the one described here; in higher-density regimes, mechanisms underlying segregation and mixing are markedly different [41, 42].

In order to further investigate the system, additional data was obtained from simulations produced using the MercuryDPM software developed at the University of Twente [43–45]. Values of N, f, A, W, H and ε_w used in simulations correspond precisely to experimental values. For intra-species particle collisions, values of effective elasticity analogous to the experimental measurements of Feitosa and Menon [37] are implemented - specifically, $\varepsilon_{glass} = 0.83$, $\varepsilon_{steel} = 0.79$ and $\varepsilon_{brass} = 0.61$. For collisions between dissimilar particles, effective elasticity is taken as $\varepsilon_{\alpha\beta} = (\varepsilon_\alpha + \varepsilon_\beta)/2$, where α and β represent individual species. This relationship arises naturally from the spring-dashpot model of particle elasticity [46–48], and is found to produce good agreement between experimental and simulational results. A collision time $t_c = 10^{-5}$ is used. Analysis of the evolution of the separate species' vertical centres of mass is used to ensure a steady state. In order to isolate the effects of inelasticity in the system, the frictional coefficient, μ, is set to 0. As can be seen in Figure 4, despite some discrepancy in the absolute values of segregation intensity, the general trend of the relationship between I_s and J_L/J_H corresponds closely to that observed in experiment, demonstrating that the effect of convection persists even in the absence of friction. Moreover, the reintroduction of μ to the system at values of 0.15, 0.5 and 1.0 was found to produce no significant variation in results, strongly suggesting that friction does not play a significant rôle in the observed behaviour. This is unsurprising considering the dilute nature of the systems investigated. Simulations were also conducted using the relevant elasticity and density values for a glass/brass system. Once again, the general trend of I_s vs. J_L/J_H was found to be consistent. However, certain differences were also observed. Firstly, as can be seen in Figure 4, the typical magnitude of I_s is considerably increased due to the greater disparity in particle masses and elas-

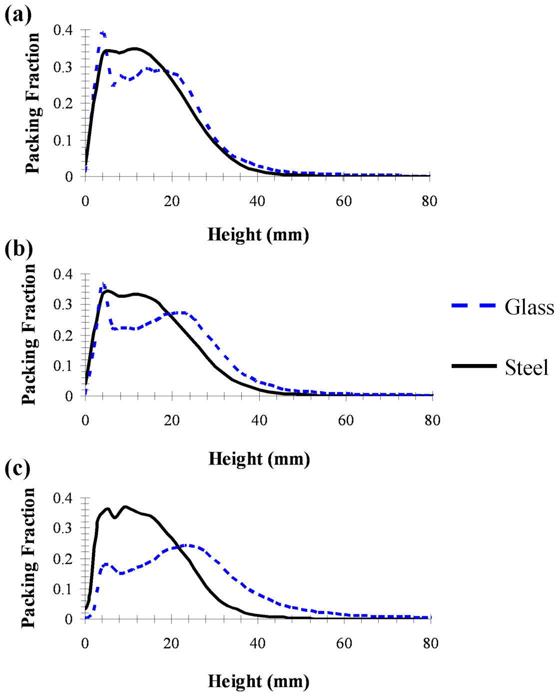

FIG. 3: Density profiles for both components of a binary glass/steel system corresponding to (a) steel ($\varepsilon_w = 0.70$) (b) perspex ($\varepsilon_w = 0.33$) and (c) brass ($\varepsilon_w = 0.58$) sidewalls.

ticities. It was also found that, for a given system, the *overall strength* of convection was considerably reduced compared to the glass/steel case. This is understandable, as the reduced elasticty of brass particles will lead to an increased average dissipation for interparticle collisions. This will lead to a reduced density gradient between the central and outer regions of the system, and hence reducedl J_{tot}. Conversely, the *difference* in flow rate between species is typically found to increase. It is worth noting that similar behaviour was observed for simulations in which the density ratio was held constant at the steel/glass value and only elasticity was varied. The findings discussed above also agree qualitatively with experimental observations of brass/glass systems (not shown) [55]. Finally, simulations were conducted using an increased system size. Figure 4 also shows data corresponding to a system for which W has been increased by a factor of 2 and N by a factor of 4, thus maintaining a consistent resting bed height and ensuring that the control parameter $F_d = H(1-\varepsilon)$ [49] is held constant. The closely corresponding behaviour of the differently-sized systems supports the assumption of a fully 3D domain, as well as providing further evidence that wall effects are not a significant factor in the system's behaviour. The slight decrease in the average value of I_s for the wider system can perhaps be explained by the decreased relative importance of horizontal segregation. The combination of the above results shows that, despite possible variations in the 'baseline' degree of segregation within the system, the ability to significantly vary I_s through alteration of convective behaviours applies for a wide range of parameters. Through the use of simulations, it is also possible to investigate more closely the *rate* at which systems achieve their equilibrium value of I_s. Despite the significant impact of J_L/J_H on the magnitude of I_s, the time, $\Delta t_{seg.}$, in which this segregation is achieved shows no clear dependence on convective behaviour - at least, any trends are indistinguishable from the inherent statistical fluctuations in $\Delta t_{seg.}$ due to the non-deterministic nature of a system's evolution towards segregation. It is, however, interesting to note the rapidity with which the dilute systems investigated here reach their steady-state distributions - in all cases, $\Delta t_{seg.} = \mathcal{O}(10\text{s})$. Comparison with the typical timescale of convection, $t_{con.} = \mathcal{O}(1\text{s})$, implies that the system reaches its equilibrated state in only a few convective cycles. It is worth noting that the approximate timescale of this surprisingly rapid segregation can be confirmed through visual observation of the experimental system.

Although a full theoretical treatment is beyond the scope of this current paper, we now attempt to provide a qualtitative explanation for the variations in segregation strength arising from differences in convective flow. Theoretical work by Garzó [15, 16] shows that, in highly fluidised yet non-convective systems, the degree of segregation reached by a granular bed is determined by a balance between separation caused by thermal diffusion and remixing due to 'ordinary' diffusion. In the system currently under investigation, however, one must also consider additional diffusive motion due to convection[50]; the lower probability of heavier particles being dragged into the convective stream [40] will lead to a reduced relative diffusivity for this species. A greater disparity in diffusivity between particle species will give a more significant imbalance between processes favouring segregation and those opposing it. Thus, for the case of highly disparate flow rates between species, their increased relative motion can be expected to create more pronounced concentration gradients and hence increased segregation [15, 16, 51]. Conversely, when convection for both species is equally strong (or, indeed, equally weak), a reduced I_s is to be expected. Support for this hypothesis can be seen in Figure 2(b), which shows a monotonically increasing I_s as the self-diffusion coefficients for the 2 species become more disparate.

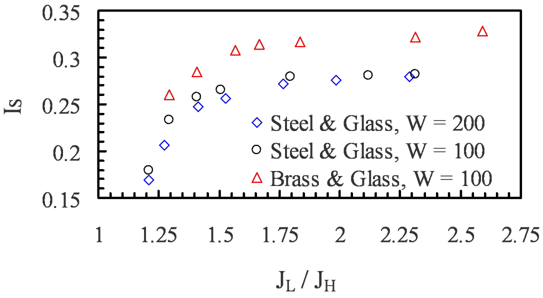

FIG. 4: Simulational data showing the variation of segregation intensity with the ratio of convective flow rates of light and heavy particles.

IV. CONCLUSIONS

Convective motion has been shown to play a crucial rôle in the segregation of a highly fluidised granular bed whose components differ in their material properties. A mechanism has been proposed to explain a previously unobserved relationship between convective flow rate and segregation intensity. It is hoped that further research stemming from this observation may lead to a theoretical framework describing segregation in the convective regime. The results of this study strongly imply the possibility that the degree of segregation within a granular bed can be controlled by altering convective behaviour through adjustment of either the driving parameters of the system *or* the material properties of the container in which it is housed. This latter method may prove particularly useful in situations where it is undesirable or unfeasible to alter the manner in which a system is driven, for example in certain industrial applications. Moreover, the ability to vary segregation intensity independently of driving force could also potentially lead to significant energy savings.

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[52] It should be noted that thermal convection, as discussed in this paper, wherein particle interactions are predominantly inertial, is distinct from the primarily frictionally-driven convection observed in less fluidised systems [23].

[53] For the sake of brevity, the athermal granular temperature of the system will, for the remainder of this paper, be referred to simply as the temperature.

[54] For further details regarding the calculation of J, please refer to reference [24]

[55] Due to experimental difficulties in creating convection within more internally dissipative systems, only a limited number of experimental data points were obtained. Nonetheless, the differences in the average magnitude of I_s, as well as the absolute and relative values of J observed in simulation were also present in experimental data. Moreover, the expected monotonic increase of I_s with J_H/J_L was observed, although the number of data points was inadequate to reliably confirm a specific trend.